REAL-TIME ALGORITHM FOR GLOBALLY OPTIMAL IMPULSIVE CONTROL OF SPACECRAFT FORMATIONS

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Proposed formation flying missions require more efficient control algorithms for challenging scenarios

- Lifetimes are limited by propellant capacity
- Missions must operate in eccentric orbits with multiple perturbations
- Multiple attitude modes result in time-varying cost of a specified maneuver

Target Star

Line-of-Sight

Miniaturized Distributed Occulter/Telescope (mDOT) mission concept
Problem Statement

- Develop a solution methodology that provides globally optimal impulsive control inputs for fixed-time, fixed-end-condition control of linear time-varying systems
- Solution based on geometric relationships between reachable sets
- Resulting algorithm accommodates:
  - Multiple state definitions
  - Multiple perturbations in eccentric orbits
  - A wide range of time-varying cost functions

CubeSat Proximity Operations Demonstration (CPOD) Mission (Image: NASA)
Geometry of Problem: Cost of Control Input

Propellant cost of a maneuver is:
1) Linear in magnitude
2) Varies with direction

Cost of control input at any specified time must be norm-like:
1) Cost proportional to $||u||$
2) Sublevel sets are convex

Example cost function sublevel sets
Geometry of Problem: Reachable Sets

- Reachable set of states for a specified cost is convex
- A cost is optimal if the target state is on the boundary of the reachable set
Solution Approach

- Primal problem: minimize cost subject to constraint that target is reachable
  - Shape of reachable set not known in advance
- Dual problem: maximize cost subject to constraint that target cannot be reached at a lesser cost
  - Equivalent to maximizing the cost to reach a plane that contains the target state

Example supporting hyperplanes
Lower Bounds on Minimum Cost

- Each feasible solution to the dual problem is a lower bound on the minimum cost.
- Each choice of $\lambda$ defines a supporting hyperplane that is tangent to the reachable set.
- Using multiple $\lambda$ it is possible to compute an arbitrarily accurate polyhedral approximation of the reachable set.

Example circumscribing polyhedra
Solution Algorithm

1. Initial $\lambda$
2. Compute optimal $\lambda$ for candidate times
3. Refine candidate times
4. Check if converged to within tolerance
   - Converged
   - Not Converged
5. Optimal control input extraction
Validation

- Solved 1000 reconfigurations based on mDOT with time-varying attitude constraints
- Solutions required to have total cost within 1% of global optimum
- Algorithm converged in 1-7 refinement iterations
- Algorithm was implemented on a Tyvak flatsat processor (800 MHz) with a run time of 3-10 seconds
Conclusions

- Developed algorithm that computes optimal impulsive maneuvers for fixed-time, fixed-end-condition control of linear time-varying systems
  - Accommodates multiple state definitions
  - Accommodates dynamics models for eccentric orbits including perturbations
  - Can capture effects of attitude constraints in cost function definition
  - Solutions have total cost within user-specified tolerance of global optimum
  - Globally convergent and insensitive to poor initializations
  - Runs in real-time on space-qualified processor
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