Quantification of Model Form Uncertainties in Eigenvalues Computation

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Outline

Generalized Eigenvalue Problem and Objectives

Stochastic Formulation for the GEVP

Application and Results

Conclusion
Compute the mode shapes and frequencies of a structure;
Mesh the structure and use Finite Element Analysis;
Stiffness and mass matrices $K \in \mathbb{M}_{N,N}$ and $M \in \mathbb{M}_{N,N}$;
The generalized EVP can be written as:
$$K u = \omega^2 M u,$$
where $u$ is the mode shape and $f = \omega/(2\pi)$ is the frequency.
$N$ is the number of degrees of freedom.
Modeling Uncertainties

- Parametric uncertainties (geometry, boundary conditions, materials properties, etc);

- Modeling uncertainties:
  - Some of the physic may not be present in our model;
  - Use of a simple model.
Modeling Uncertainties
Projection-Based Model Reduction

- Reduce the dimension of the solution to \( n << N \);
- Use a Reduce Order Basis (ROB) \( V \);
- Time domain problem:

\[
M\ddot{y} + Ky = f(t)
\]

\[
y(0) = y_0
\]

- Approximate equations:

\[
y \approx Vy_r,
\]

\[
V^TMV\ddot{y}_r + V^TKVy_r = f(t).
\]

- Reduced GEVP:

\[
K_ru_r = \omega_r^2M_ru_r,
\]

where, \( K_r = V^TKV \) and \( M_r = V^TMV \).
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Randomization of $V$ into a random matrix $W$:

- $W$ must satisfy some properties:

$$W^T M W = I_n,$$

$$B^T W = I_{n_{CD}}.$$

- $W \in \mathbb{M}_{N,n}$ is constructed using the Maximum Entropy principle;

Stochastic Generalized EVP:

$$K_r u_r = \omega_r^2 M_r u_r,$$

where, $K_r = W^T K W$ and $M_r = W^T M W$.

$W$ depends on some hyperparameters $\alpha$ calibrated using available data.

Stochastic Formulation for the GEVP
Summary of the Method

- Stochastic model \( W(\alpha) \Rightarrow \) uncertainties quantification.

- Monte-Carlo method: \( \mathbb{E}[W(\alpha)] \) and \( \text{Var}[W(\alpha)] \).
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mAEWing1 Flying Wing

- Replica of X-56 type aircraft made of composite material;
- Experimental measurements made at the University of Minnesota.
Finite Element Model

- Number of elements: 1828;
- Number of degrees of freedom: 4146;
- Model form uncertainties:
  - homogenized representation of the composite material;
  - lack of accounting for damping.
Performance of the SROM

- Basis of size $n = 10$ and $n_{\text{modes}} = 7$.
- SROM encompass the experimental data.
- No data for the fifth and seventh mode.
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Conclusions

- The results demonstrate the ability of the SROM for model form uncertainty quantification.

- This method can be used for non-linear dynamics problem.

- This method can be coupled with hyper-reduction for very fast computation.